Student Number: ____________________ **Teacher Name:** ____________________

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PENRITH HIGH SCHOOL

2016
HSC TRIAL EXAMINATION

Mathematics Extension 1

General Instructions:

- Reading time 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Start a new answer sheet for new question
- Show working for questions 11 – 14
- BOSTES Reference Sheet is provided
- Do NOT tear off any paper from this question paper

Total marks–70

SECTION I **10 marks**

- Attempt Questions 1–10
- Allow about 15 minutes for this section

SECTION II **60 marks**

- Attempt Questions 11–14
- Allow about 1 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

Assessor: Daniel Antone

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple–choice answer sheet for Questions 1–10

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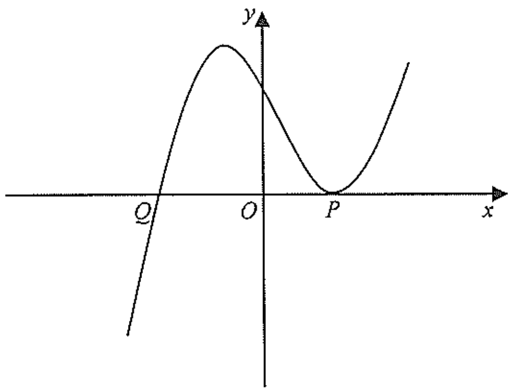
1) When the polynomial $P(x) = 2x^4 + 9x^3 - 4x - 6$ is divided by x the remainder is?

(A) 1 (B) -6 (C) 3 (D) -3

2) The polynomial function represented below shows its curve touches the x-axis at the point P and crosses it at the point Q. This polynomial then has:



(A) no roots
(B) one root only
(C) two distinct roots only
(D) one simple root and one double root only

3) The solution of the inequality $\frac{1}{1+x^2} \leq 1$ is

(A) $x \leq 1$ (B) $x \geq 1$ (C) All real numbers (D) No solution

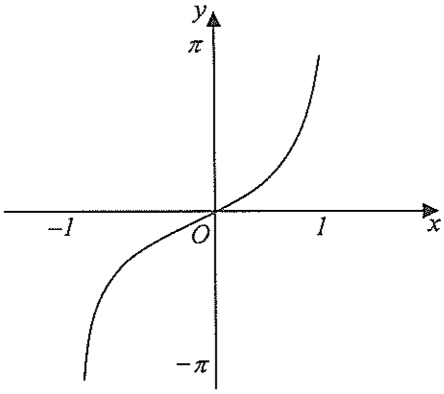
4) A point is(x, y) is given by $x = 1 + 2\theta$ and $y = 1 - 2\theta$ where θ is a real number. Hence the relation between x, y is:

(A) $x + y + 2 = 0$ (B) $x + y - 2 = 0$
(C) $x - y + 2 = 0$ (D) $x - y - 2 = 0$

5) The point P is $(1, 2)$ and Q is $(3, 4)$. The point $M(4, 5)$ divides the interval PQ into the ratio:

(A) $2 : 1$ (B) $1 : 2$ (C) $-1 : 3$ (D) $3 : -1$

6) Which of the following options could be the equation of the graph shown below?



(A) $y = \sin^{-1} x$ (B) $y = 2\sin^{-1} x$

(C) $y = \sin^{-1} \frac{x}{2}$ (D) $y = \sin^{-1} 2x$

7) A particle is moving in a Simple Harmonic Motion and its displacement, x, at time t is given by the equation $x = 2 + a\cos(nt)$. Then the CORRECT statement of the following is

(A) The particle started its motion at $x = 0$
(B) The initial phase is $\frac{\pi}{2}$
(C) The centre of motion is $x = 2$
(D) The amplitude is $2a$

8) Giving the fact that $\frac{2}{1-x^2} = \frac{1}{1+x} + \frac{1}{1-x}$ (Do NOT prove this) hence $\int \frac{2dx}{1-x^2} =$

(A) $\tan^{-1} x + C$ (B) $-\tan^{-1} x + C$

(C) $\ln\left(\frac{1+x}{1-x}\right) + C$ (D) $\ln\left(\frac{1-x}{1+x}\right) + C$

9) The term independent of x in the expression $\left(\frac{1}{x}+x\right)^{10}\left(1+x^2\right)$ is

(A) $\binom{10}{4}+\binom{10}{5}$ (B) $\binom{10}{5}+\binom{10}{5}$

(C) $\binom{12}{4}+\binom{12}{5}$ (D) $\binom{8}{5}+\binom{8}{5}$

10) A particle moves in a **SHM** with the displacement x at any time t is given by $x = a\cos(nt)$, where a and n are constants. Let b be any number such that $0 < b < \frac{a}{3}$. When the particle moves passing the point $x = b$ for the first time, the only CORRECT statement of the following is

(A) The particle has passed the centre of motion
(B) The particle is heading towards the centre of motion
(C) The particle is at rest
(D) The particle is slowing down

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

In Questions **11–14,** your responses should include relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) Start a NEW booklet **Mark**

11-a) Solve graphically the inequality $\frac{1}{1+x} \le 1+x$ **2**

11-b) Differentiate $\frac{x \sin x}{e^x}$ **2**

11-c) Use the substitution $u = 1+\sqrt{1+x^2}$, or otherwise, find $\int \frac{x\,dx}{(1+\sqrt{1+x^2})\sqrt{1+x^2}}$ **2**

11-d) Show that the general term of the expansion $\left(\frac{x^2}{2}+\frac{2}{x^2}\right)^{22}$ **3**

can be written as $T_{k+1} = \binom{22}{k} 2^{2k-22} x^{44-4k}$

then find the value of k for which $\frac{T_{k+1}}{T_k} = \frac{11}{3}$, when $x = 1$

11-e) The polynomial $P(x) = x^3 + 11x - 6$ has a root near $x = 0{\cdot}5$

i. Use one application of Newton's method to obtain another approximation of this root of the given polynomial. **2**

ii. Do you think this approximation you have just obtained is better than the given approximation of the root $x = 0{\cdot}5$. Justify your opinion with mathematical calculations. **2**

11-f) The two curves $y = ax^2 + 2$ and $y = \sqrt{x}$ intersect at a right angle. Find the value of a and the coordinates of the point of intersection. **2**

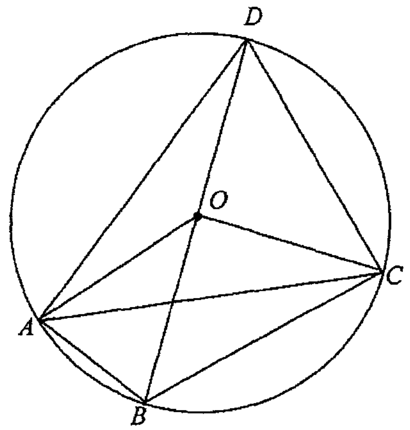
Question 12 (15 marks) Start a NEW booklet **Mark**

12-a) i. Prove that $sin^2 2x + 4cos^4 x = 4cos^2 x$ **2**

ii. Hence, or otherwise, find $\int_0^{\frac{\pi}{2}} \left(sin^2 2x + 4cos^4 x\right) dx$ **2**

12-b) Use the substitution $u = 1 - x$ to evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x}} dx$ **2**

12-c) *ABCD* is a cyclic quadrilateral prescribed in the circle *O* and *BD* is a diameter of the circle. The size of $\angle BOC$ is as **twice** as the size of $\angle AOB$.



i. Prove that $\angle BAC = \angle OCD$. **2**

ii. Prove that $\angle BOC = 4\angle OAD$. **2**

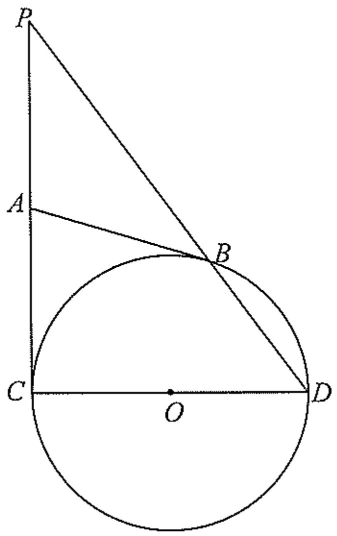
12-d) The trigonometric function $y = sin\theta - \sqrt{3}\,cos\theta$ may be written as $y = A\,sin(\theta - \alpha)$, where A and α are constants with $A > 0$ and $0 < \alpha < \frac{\pi}{2}$

i. Find the value of A and α **2**

ii. Hence, or otherwise, find the general solution of the trigonometric equation $sin\theta - \sqrt{3}\,cos\theta = \sqrt{3}$ **3**

Question 13 (15 marks) Start a NEW booklet **Mark**

13-a) A circle with centre O and radius R is shown below. From a point P outside the circle O, PC is a tangent to the circle at C. CD is a diameter and DP meets the circle at B. The point A is on PC such that AB is a tangent to the circle at B.



i. Prove that A is a centre of a circle with diameter PC **1**

ii. Show that $AB = R \tan D$ **2**

13-b) Let $f(x) = \dfrac{1}{1+e^{-x}}$.

i. State the domain and range of $f(x)$. **2**

ii. Show that the derivative of $f(x)$ can be written as $\dfrac{e^x}{(1+e^x)^2}$ **2**

iii. By considering the sign of $f'(x)$ explain why $f(x)$ has no stationary points. **1**

iv. Sketch the curve of $f(x)$ and then sketch $f^{-1}(x)$, on the same set of axes, by means of a reflection around $y = x$. Clearly indicate the intersections with axes and the asymptotes for both functions. **3**

13-c) Use mathematical induction to prove that

$2^3 + 4^3 + 6^3 + \ldots + (2n)^3 = 2n^2(n+1)^2$, for all positive integers n **4**

Question 14 (15 marks) Start a NEW booklet **Mark**

14-a) Find the general solution to the equation $\dfrac{\cos\theta - \tan\theta \sin\theta}{\cos\theta + \tan\theta \sin\theta} = -\dfrac{1}{2}$ **2**

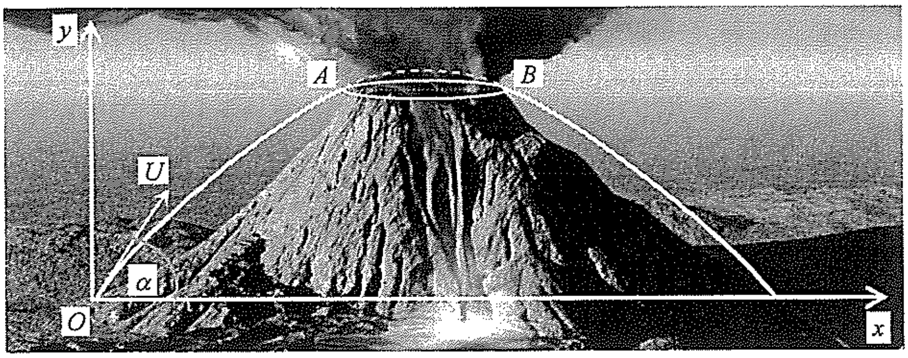
14-b) The population of a colony of a certain species of birds in an isolated island was observed for some years. It was found that the population of birds was declining as per the equation $\dfrac{dN}{dt} = k(N - 1000)$, where $N = N(t)$ is the number of birds in the colony at any time t years and k is a constant.

i. Verify that for any constant A, the expression $N(t) = 1000 + Ae^{kt}$ is the solution of the population equation. **1**

ii. The initial population was *3000* birds and the population halved after *3* years. Find, after *5* years, the number of birds in the island and calculate the reduction in population as a percentage of the initial population correct to a whole number. **3**

iii. It is known that if the number of birds in this population goes below *1100* the specie will be exposed to extinction. Assuming the living conditions in the island are not changing, when do you expect this to happen? **1**

14-c) In an attempt to estimate the diameter AB of a volcano opening and its height, a researcher, from a point O at the ground level, fired a projectile to fly over the volcano spanning the opening across its diameter A and B as shown.
The initial velocity of the projectile is U ms^{-1} at an angle α to the horizontal.



The equations of motion of the projectile may be given as

$$x = Ut\cos\alpha \text{ and } y = Ut\sin\alpha - \frac{g}{2}t^2 \text{ (Do NOT prove this)}$$

i. Show that the horizontal range R of the projectile is $\dfrac{U^2 \sin 2\alpha}{g}$ **2**

ii. The researcher fired the projectile at initial angle $\alpha = 45°$. **2**
It was observed that it landed at the other side of the volcano 100 m away from O. Show that the initial speed of the projectile is $10\sqrt{10}$ m/s (Use $g = 10$ m/s^2)

iii. It was observed that the projectile took one fifth of its flying time to span the opening (to travel from A to B) and it attained its maximum height directly above the centre of the opening as shown, show that the opening is 20 m wide. **2**

iv. How high is the volcano opening above the ground level? **2**

End of Examination

[[End Of Qns]]

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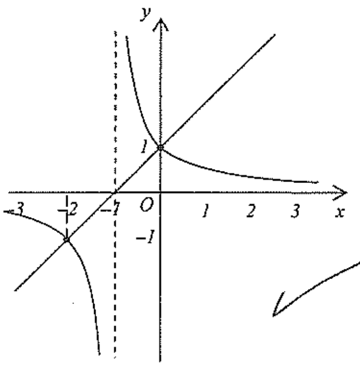
[Answers]

«1)→ B »
«2)→ D »
«3)→ C »
«4)→ B »
«5)→ D »
«6)→ B »
«7)→ C »
«8)→ C »
«9)→ A »
«10)→ B »

«11-a)→ Solving simultaneously

$y = \frac{1}{1+x}$ and $y = 1+x$

$(1+x)^2 = 1 \Rightarrow x = 0$ or $x = -2$



From the graph the solution is $x \geq 0$ or $-2 \leq x < -1$ »

«11-b)→ $\frac{e^x(x\cos x + \sin x) - xe^x \sin x}{e^{2x}}$ »

«11-c)→

$u = 1 + \sqrt{1+x^2} \Rightarrow du = \frac{xdx}{\sqrt{1+x^2}}$

$\int \frac{xdx}{(1+\sqrt{1+x^2})\sqrt{1+x^2}} = \int \frac{du}{u} = \ln u + C$ »

$= \ln\left(1+\sqrt{1+x^2}\right) + C$

«11-d)→

$T_{k+1} = \binom{22}{k}\left(\frac{2}{x^2}\right)^k\left(\frac{x^2}{2}\right)^{22-k}$

$= \binom{22}{k} 2^k x^{-2k} x^{44-2k} 2^{k-22}$

$= \binom{22}{k} 2^{2k-22} x^{44-4k}$

With $x = 1$ and $\frac{T_{k+1}}{T_k} = \frac{11}{3}$

$\Rightarrow \frac{\binom{22}{k} 2^{2k-22}}{\binom{22}{k-1} 2^{2k-20}} = \frac{11}{3}$

$\Rightarrow \frac{22!(k-1)!(23-k)!}{k!(22-k)!22!} \times 2^2 = \frac{11}{3}$

$\Rightarrow \frac{23-k}{k} = \frac{11}{12} \Rightarrow k = 12$ »

«11-e)→

i) $P(x) = x^3 + 11x - 6 \Rightarrow P'(x) = 3x^2 + 11$

$P(0\cdot5) = -0\cdot375$ & $P'(0\cdot5) = 11\cdot75$

$x_{i+1} = x_i - \frac{P(x_i)}{P'(x_i)} \Rightarrow$

$x_2 = 0\cdot5 + \frac{0\cdot375}{11\cdot75} \approx 0\cdot531915$

ii) $x_1 \approx 0\cdot5$ and $x_2 \approx 0\cdot531915$

$|P(x_1)| = |P(0\cdot5)| = 0\cdot375$ and

$|P(x_2)| = |P(0\cdot531915)| = 0\cdot0015616$

The approximation I obtained of the root is better since $|P(0\cdot531915)|$, approximately, is as 482 times closer to exact root as $|P(0\cdot5)|$. »

«11-f)→

$2ax \times \frac{1}{2\sqrt{x}} = -1 \Rightarrow a\sqrt{x} = -1 \Rightarrow x = \frac{1}{a^2}$

But at the point of intersection $ax^2 + 2 = \sqrt{x}$

$\Rightarrow a \times \frac{1}{a^4} + 2 = \frac{-1}{a} \Rightarrow 2a^3 + a^2 + 1 = 0$ »

By trial and error $a = -1$

Point of intersection is $(1, 1)$

Exam 3 Unit Trial	MATHEMATICS	: Question 12

Suggested Solutions	Marker's Comments
a) i) $\sin^2 2x + 4\cos^4 x = 4\cos^2 x$ LHS $= 4\sin^2 x\cos^2 x + 4\cos^4 x$ $= 4\cos^2 x(\sin^2 x + \cos^2 x)$ $= 4\cos^2 x$ $=$ RHS	Most students did this correctly
ii) $\int_0^{\frac{\pi}{2}} \sin^2 2x + 4\cos^4 x \, dx$ $= 4\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ $= 4\int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x)\,dx$ $= 2\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$ $= \pi$	Some Students made silly errors
b) let $u = 1-x$ when $x=0, u=1$; when $x=1, u=0$ $\frac{du}{dx} = -1$ $du = -dx$ $\int_0^1 \frac{x^2}{\sqrt{1-x}}\,dx = -\int_1^0 \frac{(1-u)^2}{\sqrt{u}}\,du$ $= \int_0^1 \frac{1-2u+u^2}{\sqrt{u}}\,du$ $= \int_0^1 \left(u^{-\frac{1}{2}} - 2u^{\frac{1}{2}} + u^{\frac{3}{2}}\right)du$ $= \left[2u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}} + \frac{2}{5}u^{\frac{5}{2}}\right]_0^1$ $= 2 - \frac{4}{3} + \frac{2}{5}$ $= \frac{16}{15}$	Some students stopped here. However, most students did get the answer. Other students got a minus

Suggested Solutions	Marker's Comments
c) i) $\angle BAC = \angle BDC$ (circum angles subtending same arc) $\angle BDC = \angle OCD$ (base angles in an isosceles ΔOCD) where $OC = OD$, (radii of same circle) Hence $\angle BAC = \angle OCD$ ii) $\angle BOC = 2\angle BAC$ (angles at the circumference subtending same arc). Hence $\angle BOC = 2\angle OAB$ But $\angle AOB = 2\angle ADB$ (angles at the circumference subtending same arc) $\angle BOC = 2\times 2\angle ADB$ $\therefore \angle BOC = 4\angle OAD$ ($\angle AOD$ and $\angle ADO$ are base angles in isosceles ΔAOD).	Students did this well.
d) i) $y = A\sin(\theta - \alpha)$ $= A\sin\theta\cos\alpha - A\cos\theta\sin\alpha$ $\sin\theta - \sqrt{3}\cos\theta \equiv A\sin\theta\cos\alpha - A\cos\theta\sin\alpha$ $A = \sqrt{a^2+b^2} = \sqrt{1^2+\sqrt{3}^2} = 2$ $\sin\alpha = \frac{\sqrt{3}}{2}$ $\alpha = \frac{\pi}{3}$ ii) $\sin\theta - \sqrt{3}\cos\theta = \sqrt{3}$ $2\sin\left(\theta - \frac{\pi}{3}\right) = \sqrt{3}$ from (i) $\sin\theta - \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\theta - \frac{\pi}{3} = \sin^{-1}\frac{\sqrt{3}}{2} + 2n\pi$, $n = 0, \pm1, \pm2, \pm3$ $\theta = \frac{\pi}{3} + \sin^{-1}\frac{\sqrt{3}}{2} + 2n\pi$, $n = 0, \pm1, \pm2, \pm3$. $= \frac{\pi}{3} + \frac{\pi}{3} + 2n\pi$ or $\theta = \frac{\pi}{3} + \frac{2\pi}{3} + 2\pi n$	Most students did this well.

$\theta = \frac{2\pi}{3} + 2\pi n$ or $\theta = \pi + 2n\pi$, $n = 0, \pm1, \pm2, \pm3 \ldots$

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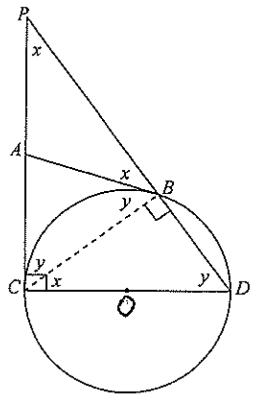
«13-a)→
i) Since CD is a diameter then $\angle CBD$ is a right angle

$\Rightarrow \angle CBP$ is a right angle and CP is a diameter

A is a centre of circle with diameter CP



ii) $PC = 2R \tan y$, but $PC = 2AB$ then $AB = R \tan D$ »

«13-b)→
i) The domain of $f(x)$ is the set of all real numbers.

To find the range take the limit as $x \to \pm\infty$ since $f(x)$ is continuous everywhere.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{1}{1+e^{-x}} = \frac{1}{1+\lim\limits_{x\to\infty} e^{-x}} = \frac{1}{1+0} = 1$$

and hence $y = 1$ is a horizontal asymptote

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\frac{1}{1+e^{-x}} = \frac{1}{1+\lim\limits_{x\to-\infty} e^{-x}} = \frac{1}{1+\infty} = 0$$

and hence $y = 0$ is a horizontal asymptote

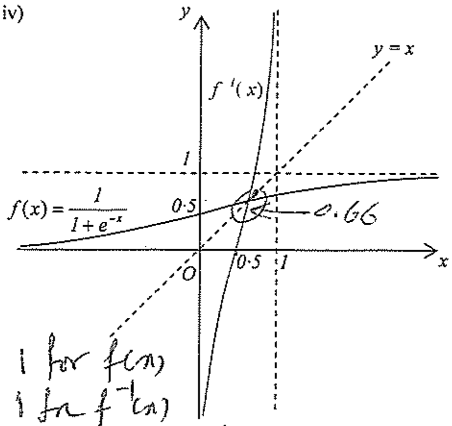
Hence the range is $0 < y < 1$ ½ ½

ii) $f(x) = \dfrac{1}{1+e^{-x}}$ apply u/v or

$$\Rightarrow f'(x) = \frac{e^{-x}}{\left(1+e^{-x}\right)^2} = \frac{e^{-x}\times e^{2x}}{e^{2x}\left(1+e^{-x}\right)^2} = \frac{e^{x}}{\left(e^{x}+1\right)^2}$$

iii) Since $e^x > 0$ for all real x, then $f'(x) > 0$ for all x, i.e. $f'(x)$ is never zero and then the function does not have stationary points.

iv)



1 for f(x)
1 for f⁻¹(x)
1 for assymptotes »

«13-c)→ Step (I): For $n = 1$

LHS $= 2^3 = 8$

RHS $= 2\times 1^2\times 2^2 = 8$ ½

$\Rightarrow$ LHS = RHS

Step (II): Assume for $n = k$, where k is a positive integer, the statement is true i.e.

$$2^3 + 4^3 + 6^3 + \ldots + (2k)^3 = 2k^2(k+1)^2$$

Step(III): Prove that, for $n = k + 1$, the statement is true

$$2^3 + 4^3 + 6^3 + \ldots + (2k)^3 + (2k+2)^3$$

$$= 2k^2(k+1)^2 + (2k+2)^3 \quad \text{From (II)}$$

$$= 2k^2(k+1)^2 + 8(k+1)^3 = 2(k+1)^2\left(k^2+4k+4\right)$$

$$= 2(k+1)^2(k+2)^2$$

Hence by the axiom of Mathematical Induction the statement is true » ½

Suggested Solutions	Marker's Comments

14 a) $\dfrac{\cos\theta - \tan\theta\sin\theta}{\cos\theta + \tan\theta\sin\theta} = -\dfrac{1}{2}$

Method 1

LHS $\dfrac{\cos\theta - \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta}{\cos\theta + \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \cos 2\theta$

$\cos 2\theta = -\dfrac{1}{2}$ ✓

$2\theta = \pi - \pi/3, \quad \pi + \pi/3$

$2\theta = 2\pi/3 \pm 2n\pi, \quad 2\theta = 4\pi/3 + 2n\pi$

$\boxed{\theta = \pi/3 + n\pi}, \quad \boxed{\theta = 2\pi/3 + n\pi}$ ✓

or $\boxed{\theta = \pm\pi/3 + m\pi}$, $m = (2n+1) \in \mathbb{Z}$, $n \in \mathbb{Z}$

~~2nd alternative solution~~

~~$2\theta = 2n\pi \pm \pi/3$~~

~~$\theta = n\pi \pm \pi/6$, $n \in \mathbb{Z}$~~

$\theta = n\pi \pm \pi/3$, $n \in \mathbb{Z}$

Method 2.

$2\cos\theta - 2\tan\theta\sin\theta + \cos\theta + \tan\theta\sin\theta = 0$

$3\cos\theta - \tan\theta\sin\theta = 0$.

$3 - \tan^2\theta = 0 \quad (\cos\theta \neq 0)$ ✓

$\tan\theta = \pm\sqrt{3}$

$\theta = \pm\pi/3 + n\pi$, ~~$\theta = 2\pi/3 + n\pi$~~ $\boxed{\begin{array}{c|c} S & A \\ \hline T & C \end{array}}$ ✓ $n \in \mathbb{Z}$

b) $N = 1000 + Ae^{kt}$

$\frac{dN}{dt} = kAe^{kt}$

(i) $= k(N-1000)$ ✓

which is true

(ii) $N(0) = 3000 = 1000 + Ae^{k0}$

$A = 2000$ $(\frac{1}{2})$

$1500 = 1000 + Ae^{3k}$

$500 = 2000e^{3k}$

$\frac{1}{4} = e^{3k}$

$3k = \ln\left(\frac{1}{4}\right) = -2\ln 2$ (1)

$k = -\frac{2}{3}\ln 2$ or $k = -0.462098$

$N = 1000 + 2000e^{5k}$

$= 1198$ birds $(\frac{1}{2})$

% reduction $= \frac{1198}{3000} \times 100$

$= 40\%$ ✓ hence reduced to 60%.

(iii) $1100 = 1000 + 2000e^{kt}$

$100 = 2000e^{kt}$

$\frac{1}{20} = e^{kt}$, $kt = \ln\left(\frac{1}{20}\right)$

$t = \ln\left(\frac{1}{20}\right) \div k = 6.48$ ✓

Suggested Solutions	Marker's Comments

(c) $x = Ut\cos\alpha$, $y = Ut\sin\alpha - \frac{g}{2}t^2$

(i) $0 = \left(U\sin\alpha - \frac{1}{2}gt\right)t$ (for ground) $\left(\frac{1}{2}\right)$

$t = 0$ or $t = \frac{2U\sin\alpha}{g}$ $\left(\frac{1}{2}\right)$ ($t = 0$ initial condition)

$x = \frac{2U^2\sin\alpha\cos\alpha}{g}$

$\text{Range}(x) = \frac{U^2\sin 2\alpha}{g}$ ✓

(ii) Range 100 m.

$\frac{U^2\sin 2\alpha}{g} = 100$, $\alpha = 45^\circ$ ✓

$U^2 = \frac{100g}{\sin 2\alpha} = \frac{100 \times 10}{1} = 1000$ ✓

$U = 10\sqrt{10}$ m/s

(iii) in $\frac{1}{5}$th flying time for the opening AB, it takes $\frac{1}{5}$th of the horizontal Range $= \frac{1}{5} \times 100$

$= 20$ m. ✓ ✓

Rk: Any mention of symmetry property is acceptable.

Exam	MATHEMATICS	: Question......14 Continued
	Suggested Solutions	Marker's Comments

(iv) $x = Ut\cos\alpha$, $y = Ut\sin\alpha - \frac{g}{2}t^2$

$t = \frac{x}{U\cos\alpha}$, $y = U\sin\alpha\left(\frac{x}{U\cos\alpha}\right) - \frac{g}{2}\left(\frac{x}{U\cos\alpha}\right)^2$

$y = x\tan\alpha - \frac{g}{2}\frac{x^2}{U^2\cos^2\alpha}$,

$\tan 45 = 1$

$\cos\alpha = \frac{1}{\sqrt{2}}$

$U^2 = 1000$

$g = 10$

$y = x - 5\frac{x^2}{1000} \times 2$

$y = x - \frac{10x^2}{1000}$

$x = 40$ (due to symmetry properties)

$y = 40 - \frac{40^2}{100}$

$\boxed{y = 24}$